

Rare Semileptonic Charm Decays

Stefan de Boer

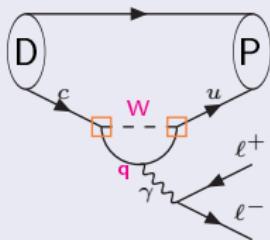
TU Dortmund

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$c \rightarrow u\ell\bar{\ell}$

FCNC, in the SM:



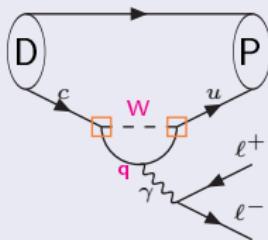
Loop-suppressed

GIM-suppressed:

$$V_{cd}V_{ud}^* + V_{cs}V_{us}^* \sim \mathcal{O}(\lambda^5) \quad \lambda \sim 0.22$$

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\rightsquigarrow Sensitive to BSM!

Complementary to rare b and s decays.

Check for theoretical tools.

Branching Fractions

High precision experiments:

(LHCb, CMS, BaBar, Belle II, CLEO-c, BESIII, ...)

Non-resonant [\[LHCb 2013\]](#) :

$$\mathcal{B}(D^+ \rightarrow \pi^+ \mu^+ \mu^-) < 7.3 \times 10^{-8} \quad @ CL=90\%$$

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Theoretical status:

$$\mathcal{B}_{D^+ \rightarrow \pi^+ \mu\mu}^{\text{nr,SM}} = 6 \cdot 10^{-12} \quad \text{[Fajfer et al., 2005]}$$

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$$\mathcal{B}_{D^+ \rightarrow X_u^+ ee}^{\text{nr,SM}} = 2 \cdot 10^{-8} \quad [\text{Burdman et al., 2002}]$$

$$\mathcal{B}_{D^+ \rightarrow X_u^+ ee}^{\text{nr,SM}} = 6.0 \cdot 10^{-10} \quad [\text{Fajfer et al., 2002, 2005}]$$

Also: [\[Paul et al., 2011\]](#)

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~~~ Redo calculation!

# Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}}|_{\{q: m_q < \mu, \ell\}} + \mathcal{L}_{\text{eff}}^{\text{weak}} :$$

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{weak}}|_{\mu \sim m_W} = & \frac{4G_F}{\sqrt{2}} \sum_{q \in \{d,s,b\}} V_{cq} V_{uq}^* \\ & \times \left( C_1(\mu) Q_1^{(q)}(\mu) + C_2(\mu) Q_2^{(q)}(\mu) \right)\end{aligned}$$

$$Q_1^{(q)} = (\bar{u}_L \gamma_{\mu_1} T^a q_L)(\bar{q}_L \gamma^{\mu_1} T^a c_L),$$

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# NNLO QCD Calculation in a Nutshell

## Wilson coefficients:

- $\mu \sim m_W$ : Match SM onto effective theory.

[In collaboration with Dirk Seidel.]

[[hep-ph/9910220](#)], [[hep-ph/0411071](#)], [[hep-ph/0504194](#)], [[hep-ph/0612329](#)]

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- $\mu \sim m_b$ : Integrate  $b$ -quark ( $\rightsquigarrow$  additional operators

$$Q_{3-6} \sim \text{Diagram } 1, \quad Q_{7,8} \sim \text{Diagram } 2, \quad Q_{9,10} \sim \text{Diagram } 3.$$

Diagrams:

- Diagram 1: A quark-gluon vertex with a gluon line labeled  $c$  and a quark line labeled  $u$ . The gluon line splits into two quark lines labeled  $u$  and  $l$ , and the quark line  $u$  splits into two gluon lines labeled  $c$  and  $l$ . A summation symbol  $\sum_q$  is at the bottom left.
- Diagram 2: A quark-gluon vertex with a gluon line labeled  $c$  and a quark line labeled  $u$ . The gluon line splits into two gluon lines labeled  $c$  and  $u$ , and the quark line  $u$  splits into two gluon lines labeled  $\gamma, g$ .
- Diagram 3: A quark-gluon vertex with a gluon line labeled  $c$  and a quark line labeled  $u$ . The gluon line splits into two quark lines labeled  $u$  and  $l$ , and the quark line  $u$  splits into two quark lines labeled  $l$ .

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- $\mu \sim m_b \rightarrow \mu \sim m_c$ : Scale via RGE.

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Matrix elements at  $\mu \sim m_c$  :

- Relate  $\langle Q_{1-6,8} \rangle \sim \langle Q_{7,9,10} \rangle$  perturbatively.

[[hep-ph/9603417](#)], [[hep-ph/0306079](#)], [[arXiv:0810.4077](#)]

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- Parametrize form factor ( $f_+$ ) via z-expansion  
(parameters fitted via  $D \rightarrow \pi \ell \nu_\ell$ ).

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# Branching Fractions

Ours (preliminary) :

$$\mathcal{B}_{D^+ \rightarrow X_u^+ ee}^{\text{nr,SM}} \approx 1 \cdot 10^{-9}, \quad \mathcal{B}_{D^+ \rightarrow X_u^+ \mu\mu}^{\text{nr,SM}} \approx 2 \cdot 10^{-10}$$

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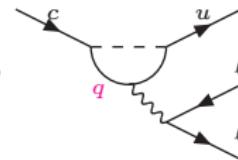
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Burdman et al. and Wang et al.:

$$C_9(\mu_W) = \sum_q V_{cq} V_{uq}^* C_{9,\text{IL}}^{(q)} \approx -0.29 \quad C_{9,\text{IL}}^{(q)} \sim$$



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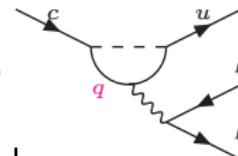
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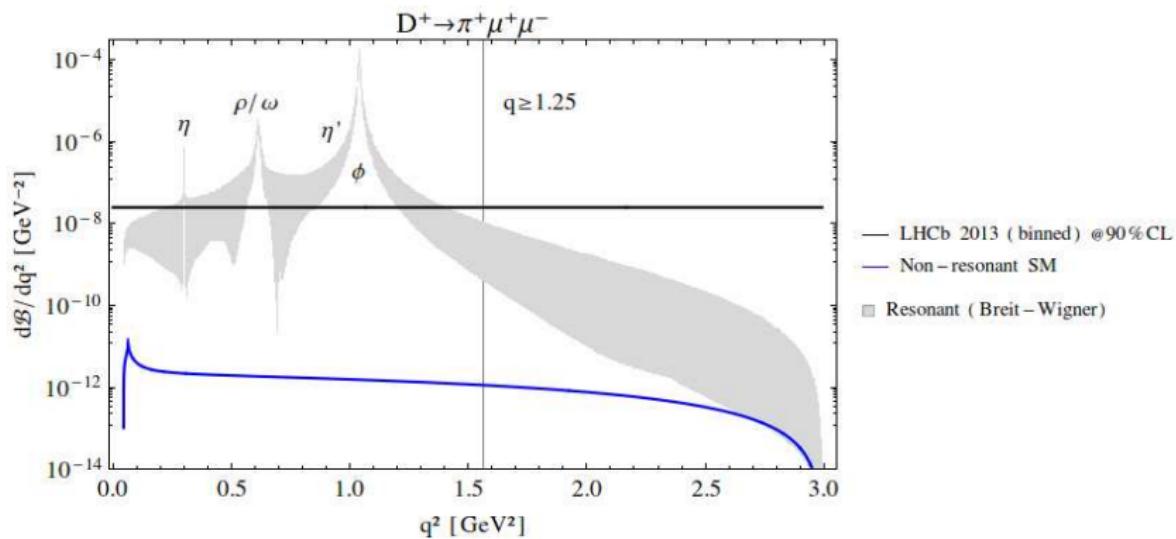
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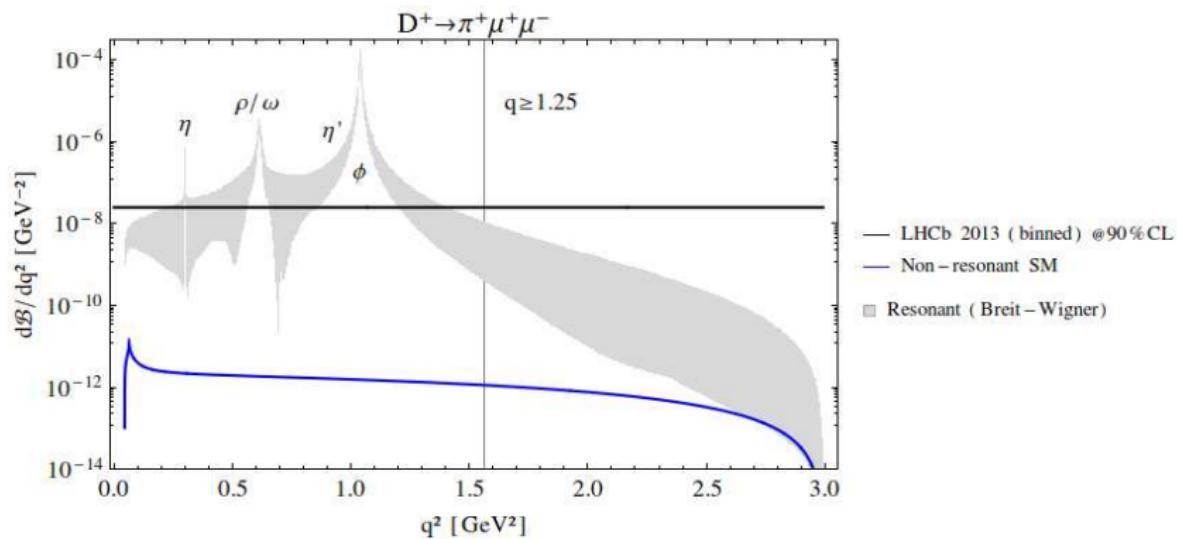
↔ Not consistent within effective theory approach.



# Decay Distribution (preliminary)



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Ours:

$$\mathcal{B}_{D^+ \rightarrow \pi^+ \mu\mu}^{\text{nr,SM}}|_{q \geq 1.25 \text{ GeV}} \approx 7.5 \cdot 10^{-13} {}^{+15\%}_{-14\%} (\bar{m}_s) {}^{+138\%}_{-46\%} (\mu_c) {}^{+27\%}_{-20\%} (f_+)$$

# BSM: Leptoquarks

Scalar (3,3,-1/3) leptoquark:  $\lambda_{S_3} (\mathbf{Q}_L^T i\tau_2 \vec{\tau} \mathbf{L}_L) \cdot \vec{S}_3^\dagger \subset \mathcal{L}_{LQ}$

Vector (3,1,-5/3) leptoquark:  $\lambda_{\tilde{V}_1} \bar{q}_R \gamma_\mu \ell_R (\tilde{V}_1^\mu)^\dagger \subset \mathcal{L}_{LQ}$

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Constrain masses via collider experiments:  $M \gtrsim 1 \text{ TeV}$

Constrain couplings via  $\mathcal{B}(D^+ \rightarrow \pi^+ e^\pm \mu^\mp)$ ,  $\mathcal{B}(D^0 \rightarrow \mu^+ \mu^-)$ ,  
 $\mathcal{B}(D^0 \rightarrow \mu^\pm e^\mp)$ ,  $x_D$ ,  $y_D$ , atomic parity violation ...

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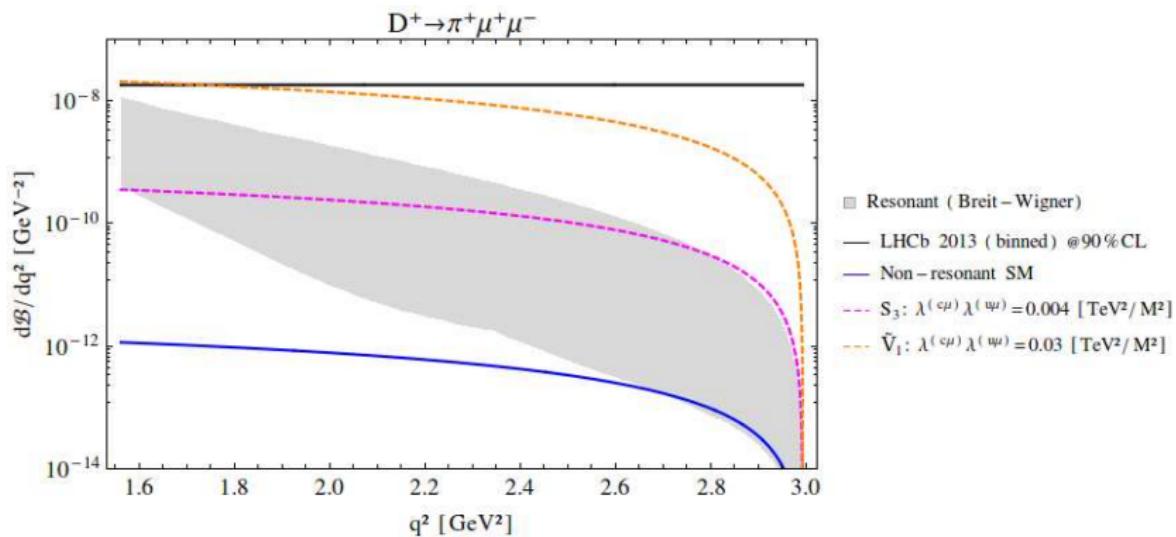
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 $\mathcal{B}(D^0 \rightarrow \mu^\pm e^\mp)$ ,  $x_D$ ,  $y_D$ , atomic parity violation ...

Apply hierarchical flavor pattern for  $S_3$  [de Medeiros Varzielas and Hiller, 2015].

## BSM: Leptoquarks (preliminary)



$$\mathcal{B}_{D^+ \rightarrow \pi^+ ll}^{S_3}|_{q \geq 1.25 \text{ GeV}} \approx 2.4 \cdot 10^{-10}$$

$$\mathcal{B}_{D^+ \rightarrow \pi^+ ll}^{\tilde{V}_1}|_{q \geq 1.25 \text{ GeV}} \approx 1.4 \cdot 10^{-8}$$

## Conclusion

$$\mathcal{B}^{\text{nr,exp}}(D^+ \rightarrow \pi^+ \mu^+ \mu^-) \lesssim 10^{-8}$$

$$\mathcal{B}_{D^+ \rightarrow \pi^+ \mu\mu}^{\text{nr,SM}} \sim 10^{-12}$$

~~ Any signal is BSM physics!? Sensitivity to leptoquarks!?

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## Outlook

- Extend this study.
- $D \rightarrow \rho \ell \ell$ ,  $\Lambda_c \rightarrow p \ell \ell$ , ...
- Look forward to experiments :)
- Let me know of your ideas!

# Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}}|_{\{q: m_q < \mu, \ell\}} + \mathcal{L}_{\text{eff}}^{\text{weak}}$$

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{weak}}|_{m_W \geq \mu > m_b} &= \frac{4G_F}{\sqrt{2}} \sum_{q \in \{d,s,\textcolor{red}{b}\}} V_{cq} V_{uq}^* \\ &\times \left( C_1(\mu) Q_1^{(q)}(\mu) + C_2(\mu) Q_2^{(q)}(\mu) \right)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{weak}}|_{m_b > \mu \geq m_c} &= \frac{4G_F}{\sqrt{2}} \sum_{q \in \{d,s\}} V_{cq} V_{uq}^* \\ &\times \left( C_1(\mu) Q_1^{(q)}(\mu) + C_2(\mu) Q_2^{(q)}(\mu) - \sum_{i=3}^{10} \textcolor{red}{C}_i(\mu) Q_i(\mu) \right)\end{aligned}$$

## SM dim 6 Operator Basis

$$Q_{1,2}^{(q)} = (\bar{u}_L \gamma_{\mu_1} \textcolor{magenta}{T^a} q_L) (\bar{q}_L \gamma^{\mu_1} \textcolor{magenta}{T^a} c_L)$$

$$Q_{3,4} = (\bar{u}_L \gamma_{\mu_1} \textcolor{magenta}{T^a} c_L) \sum_{\{q: m_q < \mu\}} (\bar{q} \gamma^{\mu_1} \textcolor{magenta}{T^a} q)$$

$$Q_{5,6} = (\bar{u}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \textcolor{magenta}{T^a} c_L) \sum_{\{q: m_q < \mu\}} (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \textcolor{magenta}{T^a} q)$$

$$Q_7 = \frac{e}{g^2} m_c (\bar{u}_L \sigma^{\mu_1 \mu_2} c_R) F_{\mu_1 \mu_2}$$

$$Q_8 = \frac{1}{g} m_c (\bar{u}_L \sigma^{\mu_1 \mu_2} T^a c_R) G_{\mu_1 \mu_2}^a$$

$$Q_{9,10} = \frac{e^2}{g^2} (\bar{u}_L \gamma_{\mu_1} c_L) (\bar{\ell} \gamma^{\mu_1} \gamma_5 \ell)$$

# Branching Fraction

Theoretical status:

$$\mathcal{B}_{D^+ \rightarrow \pi^+ \mu\mu}^{\text{nr,SM}}|_{q \geq 1.25 \text{ GeV}} = [1.18, 2.08] \cdot 10^{-10} \quad [\text{Wang et al., 2014}]$$

No scale uncertainties.

# $S_3$ Flavor Pattern

[de Medeiros Varzielas and Hiller, 2015] :

$$\lambda^{[\rho\kappa]} \sim \lambda_0 \begin{pmatrix} \rho_d \kappa & \rho_d & \rho_d \\ \rho \kappa & \rho & \rho \\ \kappa & 1 & 1 \end{pmatrix}$$

$$\rho_d \lesssim 0.02, \quad \kappa \lesssim 0.5, \quad 10^{-4} \lesssim \rho \lesssim 1, \quad \kappa/\rho \lesssim 0.5, \quad \rho_d/\rho \lesssim 1.6$$

$|\lambda_0^2 \rho \rho_d|$  constraint via rare Kaon decay and  $|\lambda_0^2 \rho| |1 - \kappa^2|$  fixed by means of  $R_K$ .